Vraag 1

(a) $$\sum_{n=0}^{\infty} \frac{1}{n^3+6} \rightsquigarrow < \underline{\frac{1}{n^3} < \frac{1}{n^2}} \quad \text{en} \quad \underline{\sum_{n=1}^{\infty} \frac{1}{n^2} < \infty}$$

(b) $$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} : \quad \underline{\int_1^{\infty} x^{-\frac{1}{2}} dx = 2x^{\frac{1}{2}} \Big|_{x=1}^{x=\infty} = \infty}$$

(c) $$\sum_{n=0}^{\infty} \frac{3n-2}{3^n} : \quad \left| \left(\frac{3(n+1)-2}{3^{n+1}}\right) \Big/ \left(\frac{3n-2}{3^n}\right) \right|$$

$$= \left| \frac{3n+1}{3^{n+1}} \cdot \frac{3^n}{3n-2} \right| = \left| \frac{1}{3} \cdot \frac{3n+1}{3n-2} \right| \rightarrow \frac{1}{3} < 1$$

(d) $$\lim_{n\to\infty} \frac{n+3}{n+7} = \lim_{n\to\infty} \frac{1+3/n}{1+7/n} = 1 \neq 0$$

(e) $$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{11}} = -1 + \frac{1}{2^{11}} - \frac{1}{3^{11}} + \frac{1}{4^{11}} - \frac{1}{5^{11}} \cdots$$

alternerend èn afzonderlijke termen worden steeds kleiner

$$\cdots < \frac{1}{(n+1)^{11}} < \frac{1}{n^{11}} < \frac{1}{(n-1)^{11}} < \cdots$$

$$T(t) = 1 - \sqrt{1+t^2} \quad \text{en} \quad \mathcal{E}(t) = e^{t^2} - \cos(t)$$

(a) $\sqrt{1+t} = (1+t)^{\frac{1}{2}} \underset{\substack{\uparrow \\ \text{formuleblad}}}{=} 1 + \frac{1}{2}t - \frac{1}{8}t^2 + \frac{1}{16}t^3 \ldots$

$$\Rightarrow \sqrt{1+t^2} = 1 + \frac{1}{2}t^2 - \frac{1}{8}t^4 + \frac{1}{16}t^6 \ldots$$

$$\Rightarrow T(t) = \underline{-\frac{1}{2}t^2 + \frac{1}{8}t^4 - \frac{1}{16}t^6 \ldots}$$

$$\mathcal{E}(t) = e^{t^2} - \cos(t) = \left(1 + t^2 + \frac{t^4}{2} + \frac{t^6}{6} \ldots\right) - \left(1 - \frac{1}{2}t^2 + \frac{1}{24}t^4 - \frac{1}{720}t^6 \ldots\right)$$

$$= \underline{\frac{3}{2}t^2 + \frac{11}{24}t^4 + \frac{121}{720}t^6 \ldots}$$

(b) $\lim\limits_{t \to 0} \frac{T(t)}{\mathcal{E}(t)} = \lim\limits_{t \to 0} \frac{-\frac{1}{2}t^2 + \frac{1}{8}t^4 - \frac{1}{16}t^6 \ldots}{\frac{3}{2}t^2 + \frac{11}{24}t^4 + \frac{121}{720}t^6 \ldots}$

$$= \lim_{t \to 0} \frac{-\frac{1}{2} + \frac{1}{8}t^2 - \frac{1}{16}t^4 \ldots}{\frac{3}{2} + \frac{11}{24}t^2 + \frac{121}{720}t^4 \ldots}$$

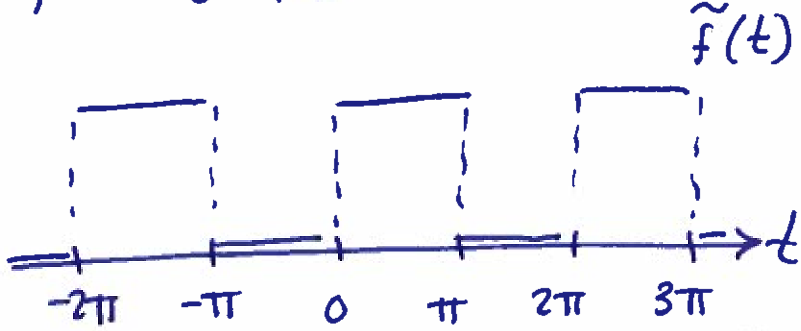
$$= {}^{-1/2}/_{3/2} = \underline{-\frac{1}{3}}$$

$$\boxed{\text{Vraag 2}}$$

Vraag 3

$$f(t) = \begin{cases} 0 & , -\pi < t < 0 \\ 4 & , 0 < t < \pi \end{cases}$$

periodieke uitbreiding:
$[f(t) \to \tilde{f}(t)]$



(a) $$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\,dt = \frac{1}{\pi}\int_{-\pi}^{0} f(t)\,dt + \frac{1}{\pi}\int_{0}^{\pi} f(t)\,dt$$

$$= \underbrace{\frac{1}{\pi}\int_{-\pi}^{\pi} 0\,dt}_{=0} + \frac{1}{\pi}\int_{0}^{\pi} 4\,dt$$

$$= \frac{4}{\pi}\, t\Big|_0^{\pi} = \underline{\underline{4}}$$

$$b_0 = \underbrace{\frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\underbrace{\sin(0)}_{=0}\,dt}_{=0} = 0$$

(b) $n > 0$: $$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos(nt)\,dt = \frac{1}{\pi}\int_{0}^{\pi} 4\cos(nt)\,dt$$

$$= \frac{4}{\pi}\,\frac{\sin(nt)}{n}\Big|_{t=0}^{t=\pi} = \frac{4}{\pi}\underbrace{\left(\frac{\sin(n\pi)}{n} - \frac{\sin(0)}{n}\right)}_{=0} = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin(nt)\,dt$$

$$= \frac{1}{\pi}\int_{0}^{\pi} 4\cdot\sin(nt)\,dt = \frac{4}{\pi}\,\frac{-\cos(nt)}{n}\Big|_{t=0}^{t=\pi} = \frac{4}{n\pi}\big(1 - \underbrace{\cos(n\pi)}_{=\pm 1}\big)$$

afhankelijk van

$$\begin{matrix} 0 & n \text{ even} \\ 8/n\pi & n \text{ oneven} \end{matrix}\Bigg\} \Leftarrow \frac{4}{n\pi}\left(1 - (-1)^n\right) \Longleftarrow$$

$$x y' = (2x+1)e^{-y}, \quad x>0$$

$$e^{y}\, dy = \frac{2x+1}{x}$$

$$\int e^{y}\, dy = \int \left(2 + \frac{1}{x}\right) dx$$

$$e^{y} = 2x + \ln(x) + c$$

$$y = \ln(2x + \ln(x) + c)$$

$$y(1) = \ln(2 + \underbrace{\ln(1)}_{=0} + c) = 0$$

$$\Rightarrow 2 + c = 1$$

$$c = -1$$

$$\Rightarrow y = \ln(2x + \ln(x) - 1)$$

$\boxed{\text{Vraag 4}}$

$\boxed{\text{Vraag 5}}$

$$\begin{cases} y'' + y' = 2e^{3t} \\ y(0) = \frac{11}{5}, \; y'(0) = \frac{18}{5} \end{cases}$$

1) $$\left.\begin{aligned} y_h'' + y_h' &= 0 \\ y_h &= e^{\lambda t} \end{aligned}\right\} \Rightarrow \lambda^2 + \lambda = 0$$

$$\lambda = 0, \; \lambda = -1$$

$$\Leftarrow \quad \boxed{y_h(t) = C_1 + C_2 e^{-t}}$$

2) $$y_p(t) = Ae^{3t}$$

$$\Rightarrow y_p'' + y_p' = 9Ae^{3t} + 3Ae^{3t} = 12Ae^{3t} \underset{\text{moet}}{=\!=} 2e^{3t}$$

$$\boxed{y_p(t) = \frac{1}{6}e^{3t}} \quad \Leftarrow \quad A = 1/6$$

3) $$y(t) = C_1 + C_2 e^{-t} + \frac{1}{6}e^{3t}$$

$$y(0) = C_1 + C_2 + \frac{1}{6} = \frac{11}{5} \quad \Rightarrow \quad C_1 + C_2 = \frac{61}{30}$$

$$\left(C_1 = \frac{61}{30} - C_2\right)$$

$$y'(t) = -C_2 e^{-t} + \frac{1}{2}e^{3t}$$

$$y'(0) = -C_2 + \frac{1}{2} = \frac{18}{5} \quad \Rightarrow \quad C_2 = \frac{1}{2} - \frac{18}{5} = -\frac{31}{10}$$

$$\Rightarrow C_1 = \frac{61}{30} + \frac{31}{10} = \frac{154}{30} = \frac{77}{15}$$

$$\Longrightarrow \underline{y(t) = \frac{77}{15} - \frac{31}{10}e^{-t} + \frac{1}{6}e^{3t}}$$

$\boxed{\text{Vraag 6}}$

$$\begin{cases} x' = yx + y \\ y' = xy + 3x \end{cases}$$

kritieke punten: $\begin{cases} y(x+1)=0 \\ x(y+3)=0 \end{cases} \Rightarrow \begin{cases} y=0 \text{ of } x=-1 \\ \text{èn} \\ x=0 \text{ of } y=-3 \end{cases}$

a)

$\Rightarrow$ twee punten: $(0,0)$ en $(-1,-3)$

b) $J = \begin{pmatrix} y & x+1 \\ y+3 & x \end{pmatrix}$

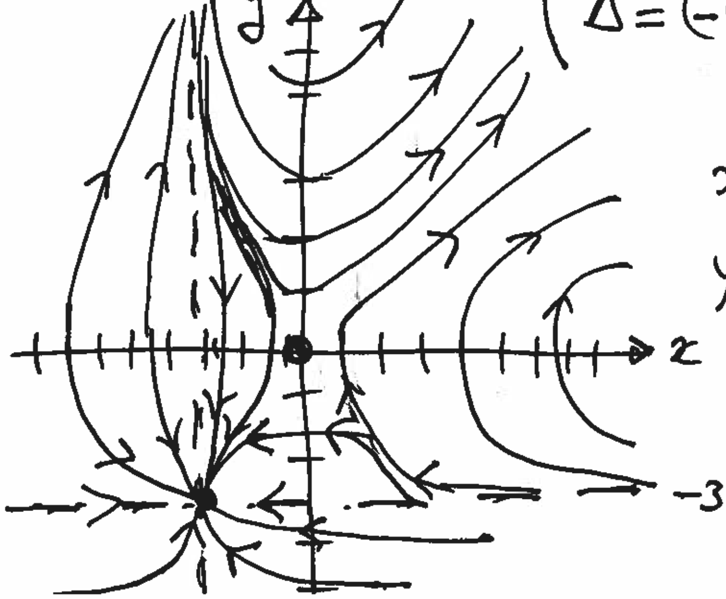
J in $(0,0)$: $\begin{pmatrix} 0 & 1 \\ 3 & 0 \end{pmatrix}$ $\left(sp(J)=0,\ det(J)=-3<0 \right)$

$\Rightarrow$ zadelpunt

J in $(-1,-3)$: $\begin{pmatrix} -3 & 0 \\ 0 & -1 \end{pmatrix}$ $\Rightarrow$ stabiele knoop

$\left(\begin{array}{l} sp(J)=-4,\ det(J)=3 \\ \Delta=(-4)^2-4*3=16-12=4>0 \end{array} \right)$

c)



$x'=0$: $y=0$ of $x=-1$

$y'=0$: $x=0$ of $y=-3$

$\boxed{\text{Vraag 7}}$

(a) $c=1$: $\quad T = (x-1)^2 + y^2 + 2022$

$$\begin{cases} \dfrac{\partial T}{\partial x} = 2(x-1) \\[2ex] \dfrac{\partial T}{\partial y} = 2y \end{cases} \Rightarrow \operatorname{grad} T\Big|_{(7,3)} = \begin{pmatrix} 12 \\ 6 \end{pmatrix}$$

$$\vec{b} = \begin{pmatrix} -2 \\ 5 \end{pmatrix} \Rightarrow |\vec{b}| = \sqrt{(-2)^2 + 5^2} = \sqrt{4+25} = \sqrt{29}$$

$$D_{\vec{b}} T\Big|_{(7,3)} = \frac{\vec{b}}{|\vec{b}|} \circ \begin{pmatrix} 12 \\ 6 \end{pmatrix} = \frac{\begin{pmatrix} -2 \\ 5 \end{pmatrix}}{\sqrt{29}} \circ \begin{pmatrix} 12 \\ 6 \end{pmatrix}$$

$$= \frac{-24+30}{\sqrt{29}}$$

$$= \frac{6}{\sqrt{29}}$$

$$T(x,y) = (x-1)^2 + c \cdot y^2 + 2022 \qquad (c \neq 0)$$

(b) $$\frac{\partial T}{\partial x} = 2(x-1) \overset{\text{stel}}{=} 0 \quad \Rightarrow x = 1$$

$$\frac{\partial T}{\partial y} = 2cy \overset{\text{stel}}{=} 0 \quad \Rightarrow y = 0 \quad (c \neq 0)$$ (*)

Dus $(1,0)$ is het enige stationaire punt

(c) $$\frac{\partial^2 T}{\partial x^2} = 2 \ , \ \frac{\partial^2 T}{\partial y^2} = 2c \quad \Rightarrow \Delta T = 2 + 2c$$

$$\frac{\partial^2 T}{\partial x \partial y} = \frac{\partial^2 T}{\partial y \partial x} = 0 \quad \Rightarrow \det(\mathcal{H}) = 4c$$

(*)

voor $c > 0 \Rightarrow \Delta T > 0$ èn $\det(\mathcal{H}) > 0$ minimum

voor $-1 < c < 0 \Rightarrow \Delta T > 0$ èn $\det(\mathcal{H}) < 0$ } zadel-

$c < 0$

voor $c < -1 \Rightarrow \Delta T < 0$ èn $\det(\mathcal{H}) < 0$ } punt

met minimale waarde van $T = 2022$ (voor alle $c > 0$)

(*)

Vraag 8

$$\begin{cases} y'' + x y' + 2y = 0 \\ y(0) = 5, \quad y'(0) = 2 \end{cases}$$

$$y(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \cdots$$

$$y'(x) = c_1 + 2c_2 x + 3c_3 x^2 + 4c_4 x^3 + \cdots$$

$$y''(x) = 2c_2 + 6c_3 x + 12 c_4 x^2 + \cdots$$

invullen $\Rightarrow$ bij x^0: $2c_2 + 2c_0 = 0$

bij x^1: $6c_3 + 3c_1 = 0$

bij x^2: $12c_4 + 4c_2 = 0$

etcetera ...

Beginconditiës $\Rightarrow$ $c_0 = 5$ en $c_1 = 2$

en dus $c_2 = -c_0 = -5$

$c_3 = -\frac{c_1}{2} = -1$

$c_4 = -\frac{c_2}{3} = +\frac{5}{3}$

Dwz $y(x) = \underbrace{5 + 2x - 5x^2 - x^3 + \frac{5}{3}x^4}_{} + \cdots$

*

Vraag 9

(a) $u(x,t) = e^{-(x-\gamma t)^2}$

$$\frac{\partial u}{\partial t} = e^{-(x-\gamma t)^2} * (-2(x-\gamma t) * \gamma)$$

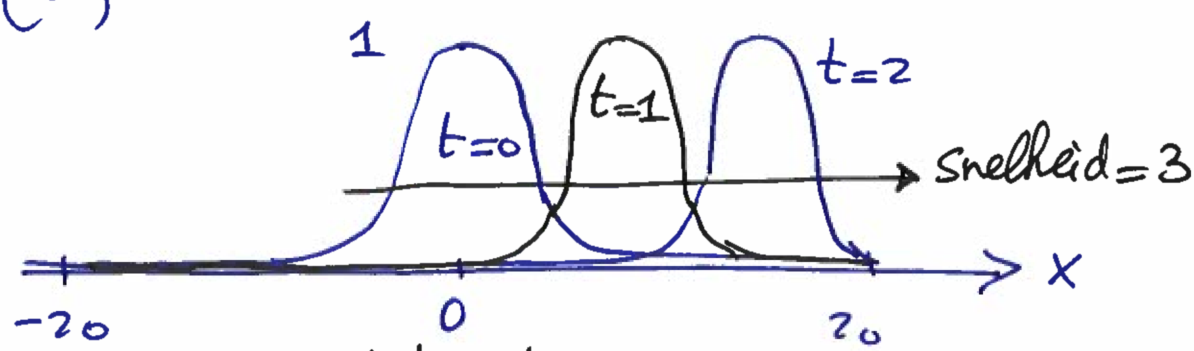
$$= -2\gamma(x-\gamma t)\, e^{-(x-\gamma t)^2}$$

$$\frac{\partial u}{\partial x} = e^{-(x-\gamma t)^2} * (-2(x-\gamma t))$$

$$\frac{\partial u}{\partial t} + 3\frac{\partial u}{\partial x} = -2\gamma(x-\gamma t)e^{-(x-\gamma t)^2} + 3\, e^{-(x-\gamma t)^2}(-2(x-\gamma t)) = 0$$

$$\Rightarrow \underline{\gamma = 3}$$

(b)



* verandert niet van vorm
* constante snelheid >0 $\Rightarrow$ verplaatsing van oplossing naar rechts

"transport"