Write down your name clearly on every sheet of paper!
You can give your answers either in Dutch or English. Good luck!

Waves test, 25 November 2015

Mathematics:

Geometrical series:

$$1 + x + x^2 + x^3 + ... + x^{N-1} = \frac{1 - x^N}{1 - x}$$

$$1 + x + x^2 + x^3 + ... = \frac{1}{1 - x}$$

$$1 - x + x^2 - x^3 + ... = \frac{1}{1 + x}$$

Taylor expansions:

$$\sqrt{1 \pm x} \approx 1 \pm \frac{1}{2} x \qquad (x \ll 1)$$

$$\sqrt{A \pm x} \approx \sqrt{A}\sqrt{1 \pm \frac{x}{A}} \approx \sqrt{A}\left(1 \pm \frac{x}{2A}\right) = \sqrt{A} \pm \frac{x}{2\sqrt{A}} \qquad (x \ll A)$$

Trigonometric relations:

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$$

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha$$

Complex exponent:

$$e^{\pm i\phi} = \cos\phi \pm i\sin\phi$$

$$e^{i\phi}e^{-i\phi} = 1$$

For correct and complete answer of each question one gets 1 point. Thus, for problem 1 one can get up to 2 points, etc.

<u>Problem 1</u>

A string of a guitar is tuned to produce an "A" note with a period of $T_1 = (1/440)$ s.

(**Question 1A**)
What is the wave velocity in the string? Assume that the string is 1 meter long.

(**Question 1B**)
What is the wavelength of the spherical sound waves produced by the guitar in air (sound velocity in air is 330 m/s)?

<u>Problem 2:</u> use trigonometric functions!

(**Question 2A**)
Using the trigonometric formulas on the first page, prove that

$$\sin x \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$$

Two waves are written as

$$A_1(x,t) = \frac{1}{2}A_0 \cos(\omega t + kx)$$

$$A_2(x,t) = A_0 \sin kx \sin \omega t$$

(**Question 2B**)

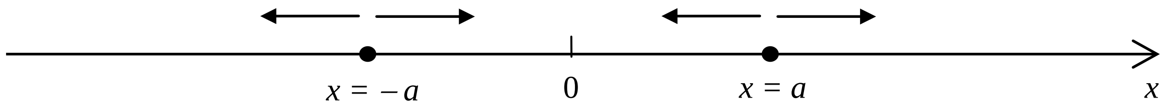
- Which of the waves, $A_1(x,t)$ or $A_2(x,t)$, is a standing wave? Motivate your answer! Calculate the positions of the nodes in this standing wave.

(**Question 2C**)
We now add the two waves together:

$$A_3(x,t) = A_1(x,t) + A_2(x,t)$$

What type of wave (travelling/standing/…) do we get as a result of their interference? Motivate your answer!

<u>Problem 3:</u> use complex notations!



On the x axis there are two identical sources of one-dimensional waves located at $x = -a$ and $x = a$.

(**Question 3A**)

Argue that the total amplitude of the wave at $x > a$ can be written as

$$A(x,t) = 2A_0\, e^{i\omega t - ikx} \cos ka$$

(**Question 3B**)
Argue that the total amplitude of the wave at $-a < x < a$ can be written as

$$A(x,t) = 2A_0\, e^{i\omega t - ika} \cos kx$$

(**Question 3C**)
Calculate the intensity of the wave $I(x)$ for $x > a$.

(**Question 3D**)
Calculate the intensity of the wave $I(x)$ for $-a < x < a$.

(**Question 3E**)
Schematically draw the $I(x)$ graph for

- $a = \lambda / 4$,
- $a = \lambda / 2$.

Use a separate sheet of paper for each part of the exam.
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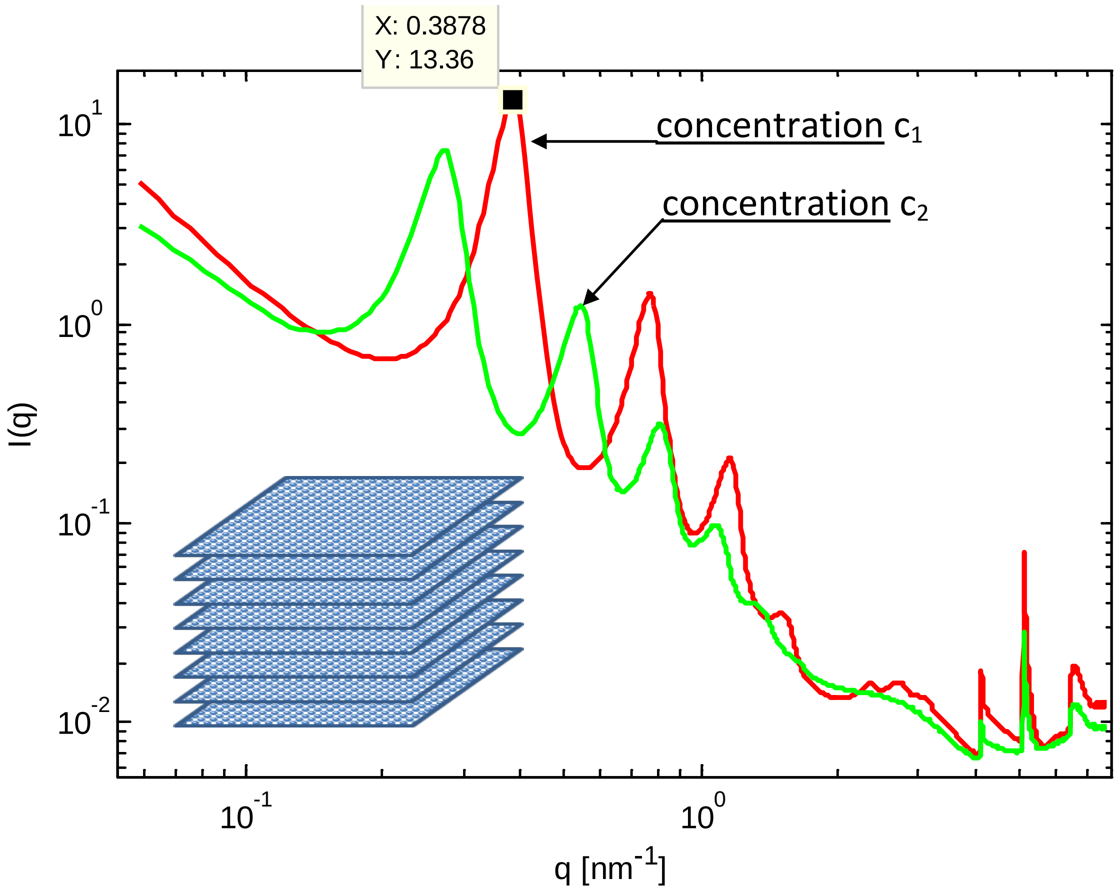
Motivate your answers clearly.

X-ray, neutron and light scattering

1. What is the meaning of the radius of gyration R_g? How can it be determined from scattering data? What range of scattering wavevectors should be used to determine R_g for particles with diameter of about 10 nm (give an order-of-magnitude estimate)? [2 points]

2. The figure below displays the small-angle x-ray scattering intensity *I*(*q*) as a function of the scattering wavevector *q* for microtubles self-assembled in solution of sodium dodecyl sulfate (SDS) and β-cyclodextrin (β-CD) in water with two different concentrations. The saw-tooth shaped peaks at scattering wavevectors q > 4 nm^{-1} are related with the 2-dimentional (2D) structure of bilayers of SDS@2β-CD complexes while the peaks at q < 4 nm^{-1} originate from periodic arrangement of the 2D bilayers into multilayer stacks as schematically illustrated in the inset. The x-ray wavelength was λ = 0.1 nm.



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a) These data are taken between q_{min} = 0.06 nm^{-1} and q_{max} = 7.5 nm^{-1}. What range of scattering angles θ was used in this experiment?
b) Does the distance between SDS@2β-CD complexes within the 2D bilayers depend on the concentration? [see data at $q > 4$ nm^{-1}]
c) Does the distance between the bilayers change with concentration? [data at $q < 4$ nm^{-1}]
d) For sample with concentration c_1 the first peak is observed at q = 0.39 nm^{-1}. What is the distance between the bilayers in the stack?
e) Is this distance larger or smaller at concentration c_2? Can you guess which of the two concentrations is higher?
f) Can light scattering be used to reveal these structures? Calculate the maximum value of the scattering wavevector q, which can be reached using visible light. Assume that the wavelength of light is λ = 633 nm (HeNe laser) and the average refractive index of the sample is n = 1.4.
g) Argue that small-angle neutron scattering using neutrons with λ = 0.3 nm can also be used to reveal these structures. Why would one like to do that? What are advantages and disadvantages of SANS in comparison with SAXS for this system?

Correct and <u>well-argumented</u> answers to questions a) – e) can give you up to 1 point each; questions f) and g) are up to 1.5 points each. The total maximum number of points for questions 1 and 2a—2g is therefore 2 + 5*1 + 2*1.5 = 10.

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Electron Microscopy [25 points max]

a. (4 points). Consider a photon with an energy of 100 eV, and an electron with an energy of 100 eV, both travelling in vacuum. For each particle, give the wavelength and the velocity. (N.B. Your answer should contain 4 numbers).

b. (5 points). Give 3 types of contrast that can generally occur when imaging in bright-field TEM imaging mode. (no explanation required)

c. (6 points) Mention the imaging mode that generates so-called Z-contrast, and explain what kind of detector has to be used for this purpose.

d. (4 points) Explain briefly how a magnet can be used to separate electrons having different energies when performing EELS.

e. (6 points) Consider the figures below, showing the results of an *in-situ* experiment whereby MgO nanoparticles react with H_2O and are transformed into $Mg(OH)_2$ upon exposure to water vapor. The circled areas in panels (a,b) show where the electron beam was during the experiment, which lasted 2400s and during which the electron beam was continuously on. From an analysis of the TEM images and of the diffraction patterns that were recorded during the experiment, discuss what happens to the crystal structure of the nanoparticles during the chemical transition, and whether the electron beam has had an influence on this process.

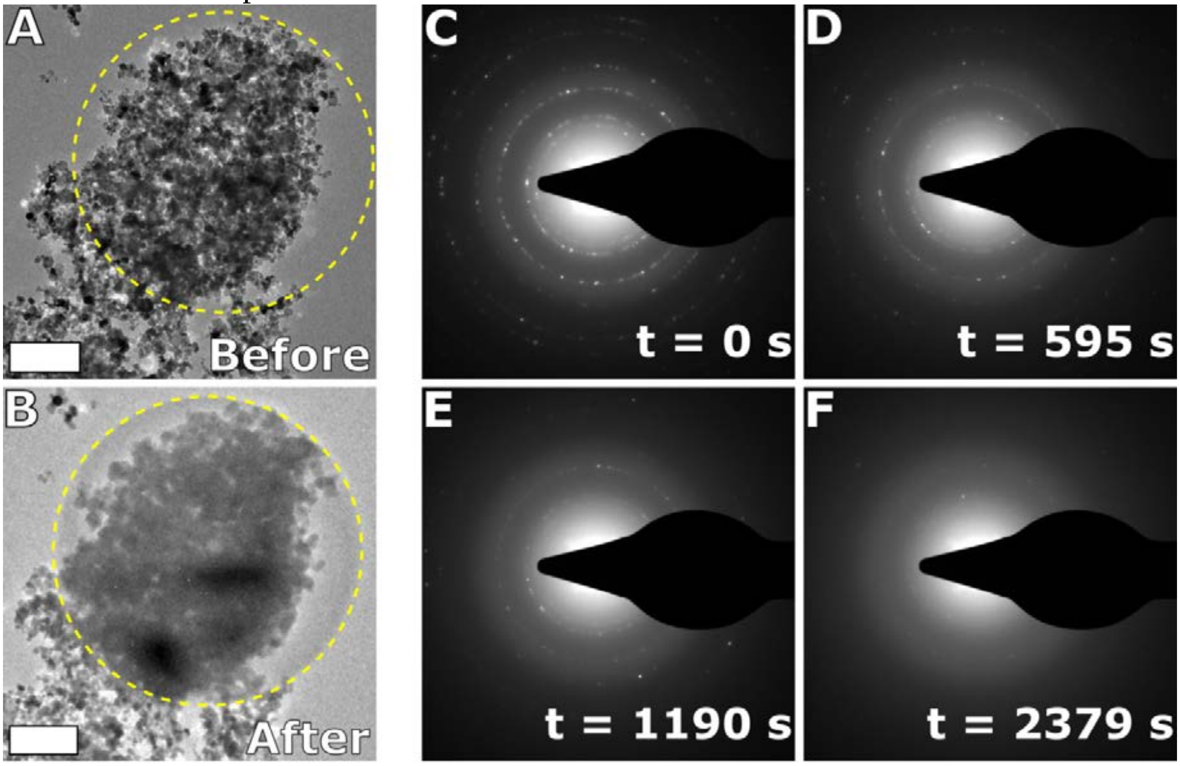

Figure. (A,B) BF-TEM images before and after the transition. The scale bars indicate 500 nm. (C-F) electron diffraction patterns at various moments in time during the transition. (Image courtesy of Wessel Vlug, Marijn van Huis).

Electron charge $e = 1.6\ 10^{-19}$ C, electron mass $m_e = 9.1 \cdot 10^{-31}$ kg, Planck constants $h = 6.6 \cdot 10^{-34}$ J·s $= 4.1 \cdot 10^{-15}$ eV·s; $\hbar = h/2\pi = 1 \cdot 10^{-34}$ J·s $= 6.6 \cdot 10^{-15}$ eV·s, speed of light in vacuum $3.0 \cdot 10^{8}$ m/s.

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Optical Microscopy

Situation 1

Alexa532 has an absorption coefficient of 85000 $cm^{-1}M^{-1}$ at 532 nm and a fluorescence lifetime of 3.8 ns. A specimen is homogeneously labelled with 10 μM Alexa532 and is studied with a confocal microscope equipped with a 60x immersion objective with an NA of 1.4. This microscope has a solid-state laser operating at a wavelength of 532 nm. The laser power at the specimen amounts to 25 μW.

a) Calculate the lateral and axial resolution of the confocal microscope.

b) Calculate how many times a single dye molecule is excited per second (excitation rate). The energy per photon can be assumed to be 3 eV.

c) Calculate the absorption cross section (in cm^2) of the dye.

d) Explain the following concepts in 5 lines max.:

- Confocal Microscope
- Airy pattern
- Chromatic aberration
- Point spread function

Situation 2

A two photon excitation microscope is equipped with a laser with a repetition rate of 82 MHz, wavelength of 1024 nm and a pulse width of 50 fs. The specimen is imaged using 5 mW of laser light. The microscope is equipped with a 60x oil immersion objective with an NA of 1.4 (similar to Situation 1).

e) Calculate the peak intensity of the laser light (in W/cm^2) in the focus of the microscope objective?

f) What is the spatial extent of the laser pulses (in m) and how many oscillations of the laser light are in a single laser pulse?

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Equations OM

Lens formula: $\frac{1}{f} = \frac{1}{v} + \frac{1}{b}$, $\frac{1}{f} = (n-1)(\frac{1}{R_1} + \frac{1}{R_2})$

Magnification: $M = \frac{b}{v}$

Numerical aperture NA = n sin(α)

Airy-disc diameter: $d_{airy} = 1{,}22\frac{\lambda}{NA}$

Resolution Rayleigh, conventional microscope: $d_{\min} = 0{,}61\frac{\lambda}{NA}$

Lateral resolution Rayleigh, confocal: $d_{\min} = 0{,}45\frac{\lambda}{NA}$

Axial resolution (FWHM) confocal: $d_{\min,ax} = 1{,}4\frac{\lambda}{n.\sin^2\alpha}$

Contrast: $\gamma = (I_{\max} - I_{\min}) / (I_{\max} + I_{\min})$

Signal-to-noise ratio: $S / N = N_{sig} / (\sqrt{N_{sig}} + D + B)$

Nyquist criterion: $N_{samples} \geq\sim 2 \cdot d_{\min}$

Absorption: $I / I_0 = \exp(-\mu C d)$

Quantum efficiency $Q = N_{em}/N_{abs}$

Energy transfer efficiency:

$E = 1 - I_{D+A}/I_D$, $E = 1 - \tau_{D+A}/\tau_D$, $E = R^6_o / (R^6_o + r^6)$, $\tau = 1/\Sigma k$ & $Q = k_{rad}/\Sigma k$

Collision quenching: $I_0 / I = \tau_0 / \tau = 1 - K_D \cdot [Q] = 1 + k_q \cdot \tau_0 \cdot [Q]$

Index of refraction oil = 1.51, index of refraction water = 1.33

Speed of light $3.00 \cdot 10^8$ m/s