<u>Write down your name clearly on every sheet of paper!</u>
You can give your answers either in Dutch or English.
Do not hesitate to ask if the question is not clear to you.

Waves Test; November 2016

Mathematics:

<u>Geometrical series</u>:

$$1 + x + x^2 + x^3 + \ldots + x^{N-1} = \frac{1 - x^N}{1 - x}$$

$$1 + x + x^2 + x^3 + \ldots = \frac{1}{1 - x}$$

$$1 - x + x^2 - x^3 + \ldots = \frac{1}{1 + x}$$

<u>Taylor expansions:</u>

$$\sqrt{1 \pm x} \approx 1 \pm \frac{1}{2} x \qquad (x \ll 1)$$

$$\sqrt{A \pm x} \approx \sqrt{A}\sqrt{1 \pm \frac{x}{A}} \approx \sqrt{A}\left(1 \pm \frac{x}{2A}\right) = \sqrt{A} \pm \frac{x}{2\sqrt{A}} \qquad (x \ll A)$$

<u>Trigonometric relations:</u>

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$$

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha$$

<u>Complex exponent:</u>

$$e^{\pm i\phi} = \cos\phi \pm i\sin\phi$$

$$e^{i\phi}e^{-i\phi} = 1$$

TOTAL maximum number of points is 100

<u>Problem 1; use trigonometric functions!</u>

Two waves are written using real functions as
$A_1(x,t) = A_0 \cos(-\omega t + kx)$
and
$A_2(x,t) = A_0 \cos(-\omega t - kx)$.

(**Question 1A, 10 points**)
Do these waves possess nodes? If yes, calculate their positions. If not, clear argue why not. Do these functions represent travelling waves? If yes, in what direction do these waves propagate and what is their velocity? If not, clear argue why not.

(**Question 1B, 10 points**)
What will be the result of the interference of the two waves $A_1(x,t) + A_2(x,t)$?
Is it a standing wave? If yes, calculate the positions of the nodes.
Is it a travelling wave? If yes, in what direction does it propagate and what is its velocity?

<u>Problem 2; use complex functions!</u>

An atom with scattering length is b is located at $x = 0$. It is irradiated by a plane wave travelling along the x-axis in the positive x-direction.

(**Question 2A, 10 points**)
Make a sketch of the situation.
Show that the amplitude of the wave along the x axis (i.e., for $y = z = 0$) can be written as

$$A(x,t) = A_0 e^{i\omega t - ikx} + A_0 \frac{b}{x} e^{i\omega t - ikx} \text{ for } x > 0 \text{ and}$$

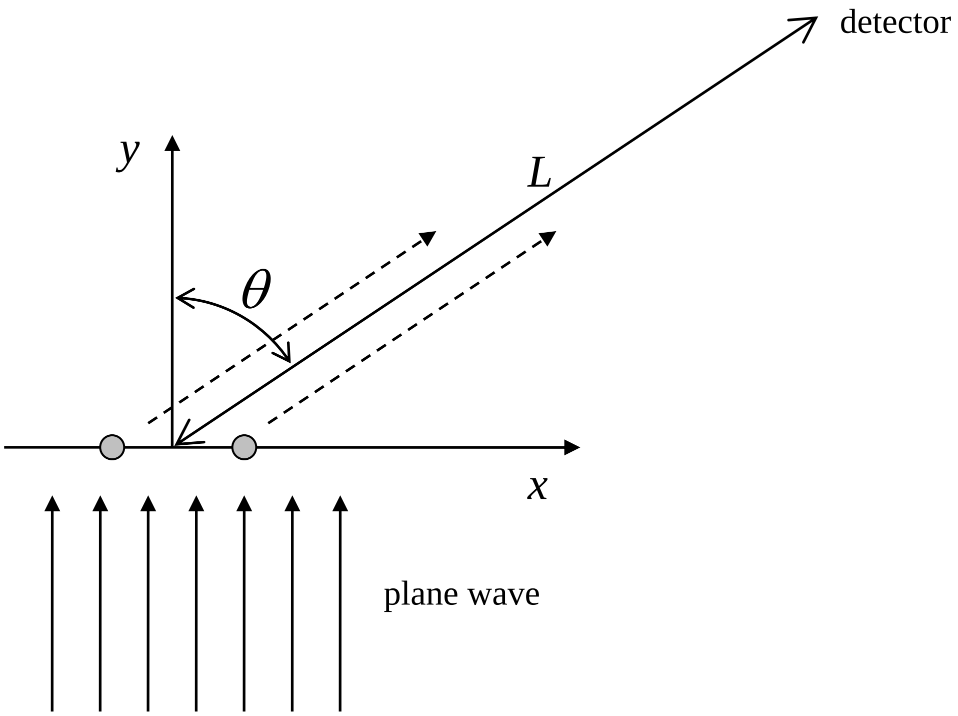
$$A(x,t) = A_0 e^{i\omega t - ikx} + A_0 \frac{b}{x} e^{i\omega t + ikx} \text{ for } x < 0.$$

Clearly explain each step of your derivation!

(**Question 2B, 10 points**)
Calculate the intensity of the wave for $x < 0$.

Problem 3; use complex functions!

Two identical point scatters are illuminated by a plane wave with wavelength λ travelling in the positive y direction. The scatters are placed on the x-axis at x = -a and x = a. A detector is placed at distance L far from the scatters (L >> 2a) such that the direction to the detector makes an angle θ with the y axis.



(**Question 3A, 10 points**)
Show that the amplitude of the wave at the detector can be written as

$$A = A_0 \frac{b}{L} e^{i\omega t - ik(L - a\sin\theta)} + A_0 \frac{b}{L} e^{i\omega t - ik(L + a\sin\theta)},$$

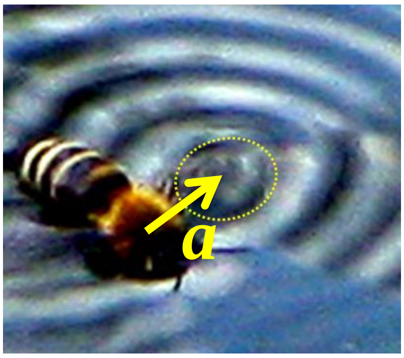
where A_0 is the amplitude of the incident wave and b is the scattering length.

Clearly explain each step of your derivation!

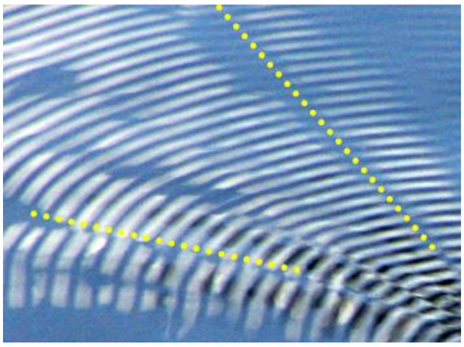
(**Question 3C, 20 points**)
Calculate the intensity of the wave at the detector. At what value of θ one observes the first minimum of the intensity if a = 50 nm and λ = 0.1 nm?

Problem 4; use complex functions!

This poor bee has felt into water and tries to fly away using its wings but the capillary forces keep it at the surface. Periodically moving wings produce waves on the water surface.



Zoom (a)



Zoom (b)

(**Question 4A, 10 points**)

Argue that a circular wave at the water surface can be written as

$$A(r,t) = \frac{B_0}{\sqrt{r}} e^{i\omega t - ikr} ,$$

where r is the distance from the source of the wave to the point of observation.

You have to explain (a) the $\sqrt{r}$ factor in the denominator and (b) the oscillating part $e^{i\omega t - ikr}$ of the wave function.

(**Question 4B, 20 points**)

In the photo given above there are two sources of the circular waves, which are the two wings of the bee. The waves produced by them can be written as

$$A(\vec{r},t) = \frac{B_0}{\sqrt{L_1}} e^{i\omega t - ikL_1} + \frac{B_0}{\sqrt{L_2}} e^{i\omega t - ikL_2} ,$$

where $L_1 = |\vec{r} - \vec{a}|$ and $L_2 = |\vec{r} + \vec{a}|$ are the distances from the point of observation to the left and the right wing of the bee. Assuming that the distance between the wings is $2a$ = 1 cm and the wavelength of the waves $\lambda = 2\pi / k$ = 5 mm, calculate in which directions there is destructive interference between the two terms in this equation.

Hint: you can assume that L_1 and L_2 are much larger than a.